ECON4925 Resource economics, Autumn 2009

Lecture 5B: Non-renewable resources: uncertainty

Standard problem

$$V(S_0) = \int_0^{\infty} e^{-et} \left[u(x) - cx\right] dt$$

Define

$$\begin{aligned} v(z,\tau) &= \int_0^{\tau} e^{-et} \left[u(x) - cx\right] dt \\ s.t.\dot{S} &= -x \\ S(0) &= S_0 \\ S(\tau) &= S_0 - z \end{aligned}$$

We have the following relationships:

If τ is exogenous and z is endogenous:

$$V(S_0) = \max_z \left[v(z,\tau) + e^{-r\tau} V(S_0 - z)\right]$$

with solution

$$v_z(z,\tau) - e^{-r\tau} V'(S_0 - z) = 0 \tag{1}$$

If τ is endogenous and z is exogenous:

$$V(S_0) = \max_\tau \left[v(z,\tau) + e^{-r\tau} V(S_0 - z)\right]$$

with solution

$$v_\tau(z,\tau) - re^{-r\tau} V(S_0 - z) = 0 \tag{2}$$

Uncertainty resolved at exogenous time τ: Together with second order condition of maximization, we can use (1) and find that uncertainty with respect to S_0 implies z down if $V''' > 0$.So initial extraction down as a consequence of uncertainty.

Uncertainty resolved at exogenous stock $S_0 - z$: Together with second order condition of maximization, we can use (2) and find that uncertainty with respect to S_0 implies τ up (since $V'' < 0$). So initial extraction down as a consequence of uncertainty.